PAM1014
Introduction to Radiation Physics

"Operations & Equations"

In this Lecture

- Introduce
  - Operations
  - Powers
  - Roots
  - Reciprocals
  - Solving Equations

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Operations

Addition (+)

- The most basic operation of arithmetic.
- Combines two numbers, the addends or terms, into a single number, the sum.
- e.g. 3 + 2 = 5
  1 + 7 = 8
  etc

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Operations

Subtraction (-)

- Essentially the opposite of addition
- e.g. 9 – 1 = 8
  1 – 1 = 0
  2 – 5 = -3
  etc

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Operations

Multiplication (x)

- Can be interpreted as repeated addition
- e.g. 7 x 3 = 21
  7 + 7 + 7 = 21

  2.1 x 3 = 6.3
  2.1 + 2.1 + 2.1 = 6.3
  etc

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Operations

Multiplication (x)

- Example:
- 3 X 5 =15
- Alternative notation

$$a \times b$$
$$a \cdot b$$
$$a\,b$$

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Operations

Division ($\div$)

- Inverse of multiplication
- Can be interpreted as repeated subtraction
- e.g. $9 \div 3 = 3$

  i.e. there are THREE 3's in 9!

- Alternative notation $\boxed{\frac{9}{3} = 3}$

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Operations

- Things worth remembering:

  $(a \times b) \times c = a \times (b \times c) = b \times (a \times c) = a \times b \times c$

  $(a + b) \times c = (a \times c) + (b \times c)$

  $(ab + cb) = (a + c) \times b$

  $(+) \times (-) = (-)$

  $(-) \times (-) = (+)$

  $(+) \div (-) = (-)$

  $(-) \div (+) = (-)$

  $(-) \div (-) = (+)$

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Common Mistakes

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Powers

- Exponentiation (or Powers) is a process generalized from repeated (or iterated) multiplication.
- The same way that multiplication is a process generalized from repeated addition.

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Powers

Positive integer exponents

- Indicates repeated multiplication by the base
- Examples:

  $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

  - 3 is the base
  - 5 is the exponent

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Examples

$2^3 =$

$3^3 =$

$5^2 =$

$10^3 =$

$10^9 =$

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Powers

Negative integer exponents

- Indicates repeated division by the base
- Example

$$3^{-5} = \frac{1}{3\times3\times3\times3\times3} = \frac{1}{243}$$

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Examples

2^{-1} =

2^{-3} =

5^{-2} =

10^{-3} =

10^{-9} =

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Reciprocals

- Any nonzero number to the (-1) power produces its reciprocal

$$x^{-1} = \frac{1}{x}; \quad x^{-n} = \left(x^n\right)^{-1} = \frac{1}{x^n}$$

- The reciprocal of a number x is the number which, when multiplied by x, yields 1.

$$\frac{1}{x}\times x = 1; \quad x^{-1}\times x = 1$$

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Reciprocals

- Note:
  - Sometimes different styles are used
  - For example,
    - Units of speed (or velocity) = m/s = ms^{-1}
    - Units of acceleration = m/s^2 = ms^{-2}

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Roots

- Square root (√)

$$If\ r^2 = x; \quad \mathrm{r} = \sqrt{\mathrm{x}}$$

- Cube root (^{3}√)

$$If\ r^3 = x; \quad \mathrm{r} = \sqrt[3]{x}$$

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Roots

- Expressing roots as powers

$$\sqrt{\mathrm{x}} = x^{\frac{1}{2}} \quad \sqrt[3]{x} = x^{\frac{1}{3}} \quad \sqrt[4]{x} = x^{\frac{1}{4}}$$

$$\sqrt[y]{x} = x^{\frac{1}{y}}$$

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Examples

$4^{1/2}$ =

$9^{1/2}$ =

$125^{1/3}$ =

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Powers

- Combinations $$x^{-\frac{1}{y}} = \frac{1}{\sqrt[y]{x}}$$
- Examples

$$9^{-\frac{1}{2}} =$$

$$8^{-\frac{1}{3}} =$$

$$1000^{-\frac{1}{3}} =$$

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Powers

- Combinations $$x^{\frac{z}{y}} = \left(\sqrt[y]{x}\right)^z$$
- Examples

$$9^{\frac{3}{2}} =$$

$$8^{\frac{2}{3}} =$$

$$1000^{\frac{2}{3}} =$$

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Powers

- Combinations $$x^{-\frac{z}{y}} = \frac{1}{\left(\sqrt[y]{x}\right)^z}$$
- Examples

$$9^{-\frac{3}{2}} =$$

$$8^{-\frac{2}{3}} =$$

$$1000^{-\frac{2}{3}} =$$

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Powers

- Important identities satisfied by exponentiation include:

$$x^{m+n} = x^m x^n$$

$$x^{m-n} = \frac{x^m}{x^n}$$

$$\left(x^m\right)^n = x^{mn}$$

- Any number to the power of zero is equal to 1 $$x^0 = 1$$

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Examples

$10^4 \times 10^6$ =

$10^{-3} \times 10^4$ =

$10^{-3} \times 10^3$ =

$10^3 \div 10^4$ =

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Solving Equations

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Solving Equations

- Equations are mathematical statements with two expressions separated by an equal sign.
- The expression on the left side of the equal sign has the same value as the expression on the right side.
- One or both of the expressions may contain variables.
- Solving an equation means manipulating the expressions to find the values of the variables.

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Solving Equations

- To keep both sides of an equation equal, we must do exactly the same thing to each side of the equation.
- If we multiply (or divide) one side by a quantity, we must multiply (or divide) the other side by that same quantity.
- If we add (or subtract) a quantity from one side, we must add (or subtract) that same quantity from the other side.

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Solving Equations

- Examples

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Background Reading

- FLAP Module M 1.1
- FLAP Module M 1.4
  - Sections 1, 2

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